Specimen Paper 9FM0/3B: Further Statistics 1 Mark scheme

Question	Scheme	Marks	AOs
1	H_0: Drivers are equally likely to be recorded speeding on any day of the week H_1: Drivers are not equally likely to be recorded speeding on any day of the week	B1	2.1
	Expected frequency = $\left[\dfrac{35+30+28+24+40+51+37}{7}\right]$	M1	3.4
	= 35	A1	1.1b
	Test statistic = $\dfrac{(35-35)^2}{35}+\dfrac{(30-35)^2}{35}+\dfrac{(28-35)^2}{35}+\ldots$	M1	1.1b
	= 13.714…	A1	1.1b
	$\nu = 7 - 1 = 6$	B1	1.1b
	$\chi^2_{(6,0.05)} = 12.592$	B1	1.1a
	In critical region, sufficient evidence to reject H_0, Significant evidence at 5% level of significance to reject Jeremy's belief.	A1	3.5a
			(8 marks)

Notes

1st B1 Both hypotheses correct (condone reference to discrete uniform distribution)
1st M1 Using uniform model to calculate expected frequencies
1st A1 35
2nd M1 Attempting to find $\sum\dfrac{(O_i-E_i)^2}{E_i}$ or $\sum\dfrac{O_i^2}{E_i}-N$ may be implied by awrt 13.7
2nd A1 awrt 13.7
2nd B1 Degrees of freedom = 6 may be implied by a correct CV
3rd B1 awrt 12.6
3rd A1 Evaluating the outcome of a model by drawing correct inference in context

Question	Scheme	Marks	AOs
2(a)	$\text{Var}(Y) = \text{E}(Y^2) - [\text{E}(Y)]^2$	M1	3.1a
	$\text{E}(Y) = \frac{1}{2}a + 2 \times \frac{3}{10} + 7 \times \frac{1}{5} [= \frac{1}{2}a + 2]$	B1	1.1b
	$\text{E}(Y^2) = \frac{1}{2}a^2 + 4 \times \frac{3}{10} + 49 \times \frac{1}{5} [= \frac{1}{2}a^2 + 11]$	B1	2.1
	$28 = \frac{1}{2}a^2 + 11 - \left(\frac{1}{2}a + 2\right)^2$	M1	1.1b
	$\frac{1}{4}a^2 - 2a - 21 = 0 \quad \rightarrow \ a = \ldots$	M1	1.1b
	$a = –6$ since $\text{E}(Y) < 0$	A1	2.2a
		(6)	
(b)	$\left(\frac{1}{3 - ('-6')}\right) \times \frac{1}{2} + \left(\frac{1}{3-2}\right) \times \frac{3}{10} + \left(\frac{1}{3-7}\right) \times \frac{1}{5}$	M1	2.1
	$= \frac{11}{36}$	A1ft	1.1b
		(2)	
			(8 marks)

Notes

(a)	1st M1 Realising that $\text{Var}(Y) = \text{E}(Y^2) - [E(Y)]^2$ is required 1st B1 Correct expression for $\text{E}(Y)$ 2nd B1 Correct expression for $\text{E}(Y^2)$ 2nd M1 Equating their expression for Var(Y) = 28 3rd M1 Solving the equation to find at least 1 value of a A1 –6 only
(b)	M1 Correct expression for $\text{E}\left(\frac{1}{3-Y}\right)$ or for finding all values of $\frac{1}{3-Y}$ A1ft $\frac{11}{36}$ or awrt 0.306 ft on $a < -4$

Question	Scheme	Marks	AOs
3(a)	$W \sim \text{Po}(0.45n)$	M1	3.1b
	$[\text{P}(W=0)=]\ \text{e}^{-0.45n} < 0.05$	M1	1.1b
	$n > 6.657\ldots$		
	$n = 7$	A1	1.1b
		(3)	
(b)	$X \sim \text{Po}(5\times0.45+5\times0.2)$ [Po(3.25)]	M1	3.3
	$\text{P}(X=2) = 0.20478\ldots$ awrt **<u>0.205</u>**	A1	1.1b
	The model is only valid if Tim and Sue make errors **independently**	B1	3.5b
		(3)	
(c)	$\text{P}(X=0) = 0.03877\ldots$	M1	3.1b
	$Y \sim \text{B}(10, \text{'}0.03877\ldots\text{'})$	M1	3.3
	$\text{P}(Y \geq 2) = 1 - \text{P}(Y \leq 1)$	M1	1.1b
	= awrt **<u>0.055</u>**	A1	1.1b
		(4)	
			(10 marks)

Notes

(a)	1st M1 Understanding that a P(0.45n) model is required here 2nd M1 For correct inequality A1 $n = 7$ cao
(b)	M1 Setting up a combined Po model A1 awrt 0.205 B1 Understanding that model is only valid if the two parts are independent
(c)	1st M1 For using Poisson distribution 2nd M1 Setting up binomial distribution 3rd M1 For finding $1 - \text{P}(Y \leq 1)$ from binomial 2nd A1 awrt 0.055

Question	Scheme	Marks	AOs
4(a)	$n = 2$ and $p = 0.6$	B1 B1	1.1b 1.1b
		(2)	
(b)(i)	P(X=1) = coefficient of t $\quad \mathrm{G}_X(t) = 0.16 + 0.48t + 0.36t^2$	M1	1.1b
	P(X=1) = **<u>0.48</u>**	A1	1.1b
		(2)	
(ii)	$\mathrm{E}(X) = \mathrm{G}'_X(1)$	M1	2.1
	$\mathrm{G}'_X(t) = 2(0.4 + 0.6t) \times 0.6$	M1	1.1b
	$\mathrm{G}'_X(1) = 1.2$	A1	1.1b
		(3)	
(c)	$\mathrm{G}_Y(t) = \mathrm{G}_X(t) \times \mathrm{G}_X(t)$		
	$\mathrm{G}_Y(t) = (0.4 + 0.6t)^4$	B1	3.1a
	$\mathrm{G}'_Y(t) = 4(0.4 + 0.6t)^3 \times 0.6$ $\mathrm{G}'_Y(1) = 2.4$	M1 A1	2.1 1.1b
	$\mathrm{G}''_Y(t) = 7.2(0.4 + 0.6t)^2 \times 0.6$ $\mathrm{G}''_Y(1) = 4.32$	M1 A1	2.1 1.1b
	$\mathrm{E}(Y^2)[= \mathrm{Var}(Y) + [\mathrm{E}(Y)]^2] = \mathrm{G}'_Y(1) + \mathrm{G}''_Y(1)$	M1	1.1b
	$\mathrm{E}(Y^2) = 2.4 + 4.32 = 6.72$ *	A1*cso	1.1b
		(7)	
			(14 marks)

Notes

(a)	1st B1 $n = 2$ 2nd B1 $p = 0.6$
(b)(i) **(b)(ii)**	M1 Finding coefficient of t A1 0.48oe 1st M1 Realising $\mathrm{G}'_X(1)$ is needed 2nd M1 Differentiation A1 1.2cao
(c)	B1 Correct use of $\mathrm{G}_Y(t) = \mathrm{G}_X(t) \times \mathrm{G}_X(t)$ 1st M1 Differentiation to find $\mathrm{G}'_Y(t)$ 1st A1 $\mathrm{G}'_Y(1) = 2.4$ 2nd M1 Differentiation to find $\mathrm{G}''_Y(t)$ 2nd A1 $\mathrm{G}''_Y(1) = 4.32$ 3rd M1 Realising $\mathrm{E}(Y^2) = \mathrm{G}'_Y(1) + \mathrm{G}''_Y(1)$ 3rd A1*cso 6.72

Question	Scheme	Marks	AOs
5(a)	$\text{H}_0 : p = 0.03$ $\quad\quad$ $\text{H}_1 : p < 0.03$	B1	2.5
	$X \sim \text{Geo}(0.03)$	M1	3.3
	$\text{P}(X \geq c) < 0.05$ $(1 - 0.03)^{c-1} < 0.05$	M1	3.4
	$c - 1 > \dfrac{\log 0.05}{\log 0.97}$	M1	1.1b
	$c > 99.35\ldots$ critical region $X \geq 100$	A1	2.2a
		(5)	
(b)	$\text{P}(X \geq 100) = 0.97^{99}$	M1	3.4
	$= \underline{\mathbf{0.0490}}$	A1	1.1b
		(2)	
(c)	Critical region $X \geq 100$ 94 is not in the critical region [$\text{P}(X \geq 94) = 0.0588\ldots > 0.05$]	M1	1.1b
	Do not reject H_0 There is insufficient evidence at the 5% level of significance that the proportion of visitors making a purchase is less than 0.03	A1	2.2b
		(2)	
			(9 marks)

Notes	
(a)	B1 Both hypotheses correct using correct notation 1st M1 Realising that the model Geo (0.03) is needed. May be implied by its use 2nd M1 Using the model to find an expression for $\text{P}(X \geq c)$ 3rd M1 Finding a valid method to solve the inequality A1 Correct critical region
(b)	M1 Using Geo(0.03) model with 100 A1 0.049 or awrt 0.0490
(c)	M1 Comparing 94 with their critical value A1 Fully correct solution and drawing a correct inference in context.

Question	Scheme	Marks	AOs
6(a)	P(Type I error) = **<u>0.05</u>**	B1	1.2
		(1)	
(b)	$\bar{X} \sim \mathrm{N}(120, \frac{3^2}{10})$ $\quad$ $\mathrm{P}(\bar{X} > c) < 0.05$	M1	3.1b
	$\dfrac{c-120}{\frac{3}{\sqrt{10}}} > 1.6449$	M1	3.4
	$c > 121.56\ldots$	A1	1.1b
	$\mathrm{P}(\bar{X} > 121.56 \mid \mu = 122)$		
	$\mathrm{P}\left(Z > \dfrac{121.56-122}{\frac{3}{\sqrt{10}}}\right) = \mathrm{P}(Z > -0.4638...)$	M1	2.1
	= 0.6786… $\quad$ = 0.68*(2sf)	A1*cso	1.1b
		(5)	
(c)	Power of Alex's test is smaller than power of Gizel's test since the null hypothesis is less likely to be rejected/Type II error has increased.	B1 B1	2.2a 2.4
		(2)	
(d)	$\dfrac{c-120}{\frac{3}{\sqrt{n}}} > 1.6449$	M1	3.4
	$c > 120 + 1.6449 \times \frac{3}{\sqrt{n}}$	A1	1.1b
	$\mathrm{P}(\bar{X} > c \mid \mu = 122) > 0.9$		
	$\dfrac{(120 + 1.6449 \times \frac{3}{\sqrt{n}}) - 122}{\frac{3}{\sqrt{n}}} < -1.2816$	M1	2.1
	$2.9265\dfrac{3}{\sqrt{n}} < 2 \quad \rightarrow \quad \sqrt{n} > 4.38...$	M1	1.1b
	$n > 19.26\ldots$ $\quad$ n = **<u>20</u>**	A1	1.1b
		(5)	
(e)	(As they both have the same size/Type I error and) Joseph's test has a higher power, so Joseph's test is recommended.	M1 A1	2.4 2.2b
		(2)	
			(15 marks)

Notes	
(a)	B1 0.05oe
(b)	1st M1 Selecting correct normal model 2nd M1 Using model to standardise and set up inequality 1st A1 Correct critical region 3rd M1 Correct probability statement to find power 2nd A1*cso awrt 0.68 with no errors seen.
(c)	B1 Correct deduction about the size of the two tests B1 Correct explanation

(d)	1st M1 Using normal model to find critical region 1st A1 Correct critical region in terms of n 2nd M1 Setting up comparison with $\|1.2816\|$ to find n 3rd M1 Solving equation to $\sqrt{n} > ...$ 2nd A1 20cao
(e)	M1 Comparison of powers A1 Correct conclusion based on power

Question	Scheme	Marks	AOs
7(a)	$[X \sim \text{NB}(12, \frac{3}{4})]$ $\binom{14}{11} \times \left(\frac{3}{4}\right)^{12} \times \left(\frac{1}{4}\right)^{3}$	M1	3.3
	= awrt **<u>0.180</u>**	A1	1.1b
		(2)	
(b)	P(X>13) = 1 – [P(X = 12) + P(X = 13)]	B1	3.1b
	$1 - \left(\left(\frac{3}{4}\right)^{12} + \binom{12}{11}\left(\frac{3}{4}\right)^{12} \times \left(\frac{1}{4}\right)\right)$	M1	1.1b
	= awrt **<u>0.873</u>**	A1	1.1b
		(3)	
(c)	$\text{E}(X) = \frac{12}{\frac{3}{4}} = 16$	M1	3.1b
	$\text{Var}(X) = \frac{12(\frac{1}{4})}{(\frac{3}{4})^2} = \frac{16}{3}$	A1	1.1b
	$\bar{X}$ □ $\text{N}\left(16, \frac{\frac{16}{3}}{30} (= 0.1\dot{7})\right)$	M1 A1ft	3.1b 1.1b
	$\text{P}(\bar{X} > 15.5) = \text{P}\left(Z > \frac{15.5 - 16}{\sqrt{0.1\dot{7}}}\right)$	M1	3.4
	$= \text{P}(Z > -1.1858\ldots)$		
	= awrt **<u>0.882/0.883</u>**	A1	1.1b
		(6)	
			(11 marks)
	Notes		
(a)	M1 Selecting correct model: negative binomial **or** B(14, $\frac{3}{4}$) with extra success A1 0.18 or awrt 0.180		
(b)	B1 Realising that P(X>13) = 1 – [P(X = 12) + P(X = 13)] M1 Correct form using negative binomial A1 awrt 0.873		
(c)	1st M1 Realising that both the mean and variance of NB are required 1st A1 Both mean and variance correct (may be implied by correct standardisation) 2nd M1 Using CLT to model $\bar{X} \sim$ N('16', …) 2nd A1ft Fully correct (or correct ft) normal distribution model for $\bar{X}$ 3rd M1 Using the normal model to find P($\bar{X}$ >15.5). Can be awarded for correct (ft) standardisation 3rd A1 awrt 0.882 or 0.883		